1. Let l_1 and l_2 be the straight lines given by $\mathbf{r}_1 = 2\mathbf{i} + \mathbf{k} + \lambda(-3\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$ and $\mathbf{r}_2 = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \mu(4\mathbf{i} + \mathbf{k})$ respectively, where λ and μ are parameters.

   (a) Show that l_1 and l_2 intersect and find the position vector of the point P of intersection.

   (b) Let α be the plane containing l_1 and l_2. Find a unit vector perpendicular to α and show that the point Q with position vector $-2\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$ does not lie on α.

   (c) Let l_3 be the line through Q parallel to l_1. Find the position vector of the point R on l_3 such that l_3 and PR are perpendicular.

2. (a) i. Show that $\mathbf{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative vector field.

   ii. Find the scalar potential.

   (b) A gas flows in a 3 dimensional environment with a constant velocity $3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and a pressure of xyz^2. Determine the rate of change of pressure the gas experiences at (1,1,2).

   (c) Determine the directional derivative of $f(x, y) = xy^2 + x^3y$ at the point (1,2) in the direction of the vector $\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$.

3. (a) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = xy - x^3 - y^2$ for each $(x, y) \in \mathbb{R}^2$.

   i. Locate all the critical points of f.

   ii. Does f have any saddle points? Justify your answer.

   iii. Show that f attains a local maximum at $(\frac{1}{6}, \frac{1}{12})$.

   iv. Is $f(\frac{1}{6}, \frac{1}{12})$ a global maximum? Justify your answer.

   (b) Let f be a real valued function defined as follows,

$$f(x, y) = \frac{x^2y}{x^2 + y^2}.$$

   i. Find the domain and range of f.

   ii. Does the limit of f at (0,0) exist? Justify your answer.

4. (a) Use Green's theorem to evaluate,

$$\oint_C x^2y dx - xy^2 dy,$$

   where C is the circle $x^2 + y^2 = 4$, going counter-clockwise.

(b) Use Stoke's theorem to calculate the line integral,

$$\oint_C y^3 dx - x^3 dy + z^3 dz.$$

The curve C is the intersection of the cylinder $x^2+y^2 = a^2$ and the plane $x+y+z = b$, where a and b are positive real numbers.

The End.